# Some Approaches to Rational Verification in Multiagent Systems

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RADICAL, 2017

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RADICAL, 2017 1 / 58

# Outline



#### Motivation

- Correctness Problem
- Nash Equilibrium
- Model Checking
- Pramework
  - Reactive Modules
  - Reactive Module Games
  - NF in RMG
  - Decision Problems
  - Complexity
- Existing Tools 3
  - MCMAS
  - FAGLE
- Ongoing and Future Works
  - EAGLE with BDD CTL SAT
  - EVE
  - NE via Parity

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#### **Classical Verification**

#### • Given a system P and formal specification $\varphi$

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### Classical Verification

- Given a system P and formal specification  $\varphi$
- Correctness: Does the behaviour of P reflect  $\varphi$ ?

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• How do we define correctness in multiagent systems?

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5 / 58

- How do we define correctness in multiagent systems?
- Each agent has her own goal, and the goals are not necessarily aligned

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5 / 58

- How do we define correctness in multiagent systems?
- Each agent has her own goal, and the goals are not necessarily aligned
- Unlike classical verification, there is no single "litmus test" for system correctness

• Agents are rational

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- Agents are rational
- Agents pursue their interests strategically

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- Agents are rational
- Agents pursue their interests strategically
- An appropriate framework for studying strategic interaction between self-interested agents: game theory

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• Many solution concepts have been proposed

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- Many solution concepts have been proposed
- Nash Equilibrium (NE) is the most widely-used

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- A player moving away from NE will be worse off

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- Nash Equilibrium (NE) is the most widely-used
- A player moving away from NE will be worse off
- Moving away (from NE) is irrational

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• A system P into a finite state model (e.g. Kripke structure)



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• Efficient model checking algorithm for CTL exists

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- LTL model checking is more complex (PSPACE-c)

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- LTL model checking is more complex (PSPACE-c)
- Symbolic MC with BDDs allows very big number of states
- Active research and development

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11 / 58

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#### • SRML is a strict subset of Reactive Module Language (RML)

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- SRML is a strict subset of Reactive Module Language (RML)
- A module in SRML consist of:

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  - interface: name, list of controlled Boolean variables

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- A module in SRML consist of:
  - interface: name, list of controlled Boolean variables
  - guarded commands: defines choices available at each state

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Formally, an SRML module  $m_i = (\Phi_i, I_i, U_i)$ , where:

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- U<sub>i</sub> is a finite set of update guarded commands s.t. ∀g ∈ U<sub>i</sub>, ctr(g) ⊆ Φ<sub>i</sub>

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module toggle controls x init ::  $\top \rightsquigarrow x' := \top$ ; ::  $\top \rightsquigarrow x' := \bot$ ; update ::  $\neg x \rightsquigarrow x' := \top$ ; ::  $x \rightsquigarrow x' := \bot$ ;

Figure 2: Example of module toggle in SRML.

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• An SRML arena  $A = (N, \Phi, m_1, \dots, m_n)$ 

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- $N = \{1, \dots, n\}$  a set of agents
- $m_i = (\Phi_i, I_i, U_i)$
- $\{\Phi_1, \ldots, \Phi_n\}$  is a partition of  $\Phi$

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Reactive Modules

#### Reactive Module Games

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• RMG 
$$G = (A, \gamma_1, \ldots, \gamma_n)$$

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RMG G = (A, γ<sub>1</sub>,..., γ<sub>n</sub>)
A = (N, Φ, m<sub>1</sub>,..., m<sub>n</sub>)

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- RMG  $G = (A, \gamma_1, \dots, \gamma_n)$
- $A = (N, \Phi, m_1, \ldots, m_n)$
- $\gamma_i$  is the goal (given by a temporal logic formula) of player *i*

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- $\gamma_i$  in LTL, CTL formula for LTL, CTL RMG, respectively

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18 / 58



• A *strategy* tells what action should be taken by a player in each possible situation

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  - memoryless:  $\sigma_i : V_{-i}^t \to V_i^t$  (LTL RMG),  $\sigma_i : V_{-i}^t \to 2^{V_i^t}$  (CTL RMG)

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19 / 58

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  - memoryful:  $\sigma_i : V_{-i}^{[0,t]} \to V_i^t$  (LTL RMG),  $\sigma_i : V_{-i}^{[0,t]} \to 2^{V_i^t}$  (CTL RMG)

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- Strategy in LTL RMG is deterministic, CTL RMG non-deterministic

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## Strategy Profile

• A strategy profile  $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$ 

# Strategy Profile

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- LTL RMG:  $\vec{\sigma}$  induces a run (an infinite word)  $\rho(\vec{\sigma})$

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- A strategy profile  $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$
- LTL RMG:  $\vec{\sigma}$  induces a run (an infinite word)  $\rho(\vec{\sigma})$
- CTL RMG:  $\vec{\sigma}$  induces a Kripke structure  $K_{\vec{\sigma}}$

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20 / 58

### Ex: P2P Protocol

Consider a P2P protocol with two peers: Alice and Bob

• At each time-step peers either tries to download or upload

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### Ex: P2P Protocol

Consider a P2P protocol with two peers: Alice and Bob

- At each time-step peers either tries to download or upload
- In order for one peer to download successfully, the other must be uploading at the same time
- Both peers are interested in downloading infinitely often

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## P2PP in RMG

• 
$$G_{P2P} = (A, \gamma_a, \gamma_b)$$

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## P2PP in RMG

## P2PP in RMG

## P2PP Arena in SRML

- module  $m_a$  controls  $u_a, d_a$  init
  - $:: \top \rightsquigarrow u'_a := \top, d'_a := \bot;$  $:: \top \rightsquigarrow u'_a := \bot, d'_a := \top;$

### update

$$\begin{array}{l} :: \top \rightsquigarrow u'_{a} := \top, d'_{a} := \bot; \\ :: \top \rightsquigarrow u'_{a} := \bot, d'_{a} := \top; \end{array}$$

- module  $m_b$  controls  $u_b, d_b$  init
  - $\begin{array}{l} :: \top \rightsquigarrow u'_b := \top, d'_b := \bot; \\ :: \top \rightsquigarrow u'_b := \bot, d'_b := \top; \\ \textbf{update} \end{array}$
  - $\begin{array}{l} :: \top \rightsquigarrow u'_b := \top, d'_b := \bot; \\ :: \top \rightsquigarrow u'_b := \bot, d'_b := \top; \end{array}$

Figure 3: P2PP arena in SRML.

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### P2PP Structure



Figure 4: The structure of P2PP arena.

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• 
$$\vec{\sigma} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n)$$

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$$\vec{\sigma} = (\sigma_1, \dots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \dots, \sigma_n)$$
  
•  $\vec{\sigma}' = (\sigma_1, \dots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \dots, \sigma_n)$ 

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$$\vec{\sigma} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n)$$

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$$\vec{\sigma}' = (\sigma_1, \ldots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \ldots, \sigma_n)$$

• Define preference relation  $\succeq_i$ :

$$\vec{\sigma} \succeq_i \vec{\sigma}'$$
 iff  $\vec{\sigma}' \models \gamma_i$  implies  $\vec{\sigma} \models \gamma_i$ .

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- $\vec{\sigma} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n)$
- $\vec{\sigma}' = (\sigma_1, \ldots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \ldots, \sigma_n)$
- Define preference relation  $\succeq_i$ :

$$\vec{\sigma} \succeq_i \vec{\sigma}'$$
 iff  $\vec{\sigma}' \models \gamma_i$  implies  $\vec{\sigma} \models \gamma_i$ .

• A strategy profile  $\vec{\sigma}$  is said to be a **Nash equilibrium** of *G* if for all players *i* and all strategies  $\vec{\sigma}' = (\sigma_1, \ldots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \ldots, \sigma_n)$  we have  $\vec{\sigma} \succeq_i \vec{\sigma}'$ 

- $\vec{\sigma} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n)$
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- Write NE(G) for the set of pure strategy Nash equilibria

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3

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## **NE-Emptiness**

**Problem** (NE-EMPTINESS)

Given a multiagent system G. Is it the case that  $NE(G) \neq \emptyset$ ?

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### **NE-Emptiness**

• Obviously,  $NE(G_{P2P}) \neq \emptyset$  is true



## **NE-Emptiness**

- Obviously,  $NE(G_{P2P}) \neq \emptyset$  is true
- A run that visits  $s_1$  and  $s_2$  infinitely often



### E-Nash

### Problem (E-NASH)

Given a multiagent system G and temporal formula  $\varphi$ . Is it the case that  $\rho(\vec{\sigma}) \models \varphi$  in any  $\vec{\sigma} \in NE(G)$ ?

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30 / 58

### E-Nash

• Let  $\varphi = \mathbf{GF}(d_a \wedge u_b)$ 



### E-Nash

• Let 
$$\varphi = \mathbf{GF}(d_a \wedge u_b)$$
  
•  $\exists \vec{\sigma} \in NE(G_{P2P}).\rho(\vec{\sigma}) \models \varphi$  is true



### A-Nash

• Let  $\varphi = \mathbf{GF}(d_a \wedge u_b)$ 



### A-Nash


#### **NE-Membership**

#### **Problem** (NE-MEMBERSHIP)

# Given a multiagent system G and strategy profile $\vec{\sigma}$ . Is it the case that $\vec{\sigma} \in NE(G)$ ?

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#### **NE-Membership**

• Let  $\vec{\sigma} = (\sigma_a, \sigma_b)$  where each  $\sigma_i$  prescribes only download



#### **NE-Membership**

• Let  $\vec{\sigma} = (\sigma_a, \sigma_b)$  where each  $\sigma_i$  prescribes only download • Then  $\vec{\sigma} \in NE(G)$  is true



590

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3

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## Computational Complexity Results

	LTL RMG	CTL RMG
NE-Emptiness	2EXPTIME-c	2EXPTIME-hard
E-Nash	2EXPTIME-c	2EXPTIME-hard
A-Nash	2EXPTIME-c	2EXPTIME-hard
NE-Membership	PSPACE-c	2EXPTIME-c

Table 1: Overview of computational complexity results [Gutierrez et al., 2017]

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36 / 58

## Outline

#### Motivation

- Correctness Problem
- Nash Equilibrium
- Model Checking

#### 2 Framework

- Reactive Modules
- Reactive Module Games
- NE in RMG
- Decision Problems
- Complexity
- Existing ToolsMCMAS
  - EAGLE
- Ongoing and Future Works
  - EAGLE with BDD CTL SAT
  - EVE
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• MCMAS [Lomuscio et al., 2015] uses interpreted systems [Fagin et al., 1995] for representation

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- Latest implementation supports ATL and SL

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38 / 58

- MCMAS [Lomuscio et al., 2015] uses interpreted systems [Fagin et al., 1995] for representation
- Uses global and local states to capture epistemic properties
- Latest implementation supports ATL and SL
- SL quite expressive, possible to reason about NE

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Pros:

• Has been around for more than 10 years

Cons:

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- Support GUI

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- Symbolic model checking with BDDs

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Current implementation (SLK) only supports memoryless strategy

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39 / 58

Verbosity of ISPL

Pros:

- Has been around for more than 10 years
- Support GUI
- $\bullet$  Written in a fast language (C/C++)
- Multiplatform
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Cons:

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39 / 58

- Verbosity of ISPL
- No direct support for rational verification

## Outline

#### 1 Motivation

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• EAGLE [Toumi et al., 2015] is a prototype tool for equilibrium checking

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- Accepts CTL as specification language

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41 / 58

- EAGLE [Toumi et al., 2015] is a prototype tool for equilibrium checking
- Particularly solves NE-MEMBERSHIP
- Accepts CTL as specification language
- Needs two inputs: system (modelled as CTL RMG)  $G = (A, \gamma_1, \dots, \gamma_n)$  and strategy profile  $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$

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#### EAGLE's basic algorithm:

**1** Build  $K_{\vec{\sigma}}$ 

<sup>1</sup>calling a CTL model checker oracle <sup>2</sup>calling a CLT SAT oracle;  $A_{CTL}$  is CTL representation of  $A = (N, \Phi, m_1, \dots, m_n) \circ \circ \circ$ 

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42 / 58

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EAGLE's basic algorithm:

- Build  $K_{\vec{\sigma}}$
- If ∃*i* ∈ N s.t.  $(K_{\vec{\sigma}} \not\models \gamma_i)^1$  and  $Sat(A_{CTL} \land \gamma_i)^2$  returns true, then output "NO"; otherwise "YES"

<sup>1</sup>calling a CTL model checker oracle <sup>2</sup>calling a CLT SAT oracle;  $A_{CTL}$  is CTL representation of  $A = (N, \Phi, m_{1,\overline{2}} \dots, m_{n}) \circ \circ \circ$ 

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42 / 58

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Pros:

• Uses SRML which is quite compact

Cons:

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Pros:

- Uses SRML which is quite compact
- Strategies are not memoryless

Cons:

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- Uses SRML which is quite compact
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43 / 58

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## EAGLE with BDD CTL SAT

• As reported in [Toumi et al., 2015], CTL SAT subroutine is the bottleneck

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## EAGLE with BDD CTL SAT

- As reported in [Toumi et al., 2015], CTL SAT subroutine is the bottleneck
- Can we check CTL SAT symbolically with BDD [Marrero, 2005]?

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## EVE





Figure 6: Inject RMG directly into SL BDD symbolic model checker.

RADICAL, 2017 48 / 58

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# The Punisher(s)

#### Lemma ([Gutierrez et al., 2015])

 $\rho$  is sustained by a Nash equilibrium strategy profile iff every player j whose goal is not satisfied by  $\rho$  is punishable at  $\rho$ 

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50 / 58

# The Punisher(s)

#### Nash equilibrium = Punishability + Memory

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• Memoryless determinacy

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- Memoryless determinacy
- Solves the problem of keeping track deviating run

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- Solves the problem of keeping track deviating run
- Finite number of memoryless strategies

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- Memoryless determinacy
- Solves the problem of keeping track deviating run
- Finite number of memoryless strategies
- Development of algorithms to solve PG (latest: quasipolynomial ([Calude et al., 2017], best paper award STOC 2017))

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#### Matches theoretical bound of 2EXPTIME for LTL RMGs

<sup>3</sup>ongoing joint work: Julian Gutierrez, Giuseppe Perelli, Michael Wooldridge E Saco Muhammad Najib (University of Oxford) Some Approaches to Rational Verification in L RADICAL, 2017 53 / 58

- $G_{LTL} \Rightarrow G_{PAR}$
- Compute punishing region PUN

#### Matches theoretical bound of 2EXPTIME for LTL RMGs

<sup>3</sup>ongoing joint work: Julian Gutierrez, Giuseppe Perelli, Michael Wooldridge E Saco Muhammad Najib (University of Oxford) Some Approaches to Rational Verification in L RADICAL, 2017 53 / 58

- $G_{LTL} \Rightarrow G_{PAR}$
- Compute punishing region PUN
- $G_{PAR} \Rightarrow G_{PAR}^{W}$  from  $PUN_{N \setminus W}, W \subseteq N$  (winning coalition)

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- $G_{LTL} \Rightarrow G_{PAR}$
- Compute punishing region PUN
- $G_{PAR} \Rightarrow G_{PAR}^{W}$  from  $PUN_{N \setminus W}, W \subseteq N$  (winning coalition)
- If  $\exists W$  win in  $G_{PAR}^W$ , then yes; otherwise no

Matches theoretical bound of 2EXPTIME for LTL RMGs

<sup>3</sup>ongoing joint work: Julian Gutierrez, Giuseppe Perelli, Michael Wooldridge E ၁ ۹ ۹ Muhammad Najib (University of Oxford) Some Approaches to Rational Verification in L RADICAL, 2017 53 / 58

• Consider a network composed of 2 clients: *client<sub>a</sub>*, *client<sub>b</sub>* and 2 servers: *server*<sub>1</sub>, *server*<sub>2</sub>

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54 / 58

• server<sub>1</sub> is an old server, it needs longer rest time

- Consider a network composed of 2 clients: client<sub>a</sub>, client<sub>b</sub> and 2 servers: server<sub>1</sub>, server<sub>2</sub>
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- *client<sub>b</sub>* doesn't handle urgent task, no need to be served immediately
- server<sub>1</sub> is an old server, it needs longer rest time
- *server*<sub>2</sub> needs shorter rest time

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• 
$$\gamma_a = \mathbf{G}(r_a \rightarrow \mathbf{X}(s_1 \lor s_2))$$

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$$\gamma_a = \mathbf{G}(r_a \rightarrow \mathbf{X}(s_1 \lor s_2))$$
  
•  $\gamma_b = \mathbf{G}(r_a \rightarrow \mathbf{F}(s_1 \lor s_2))$ 

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•  $\gamma_1 = \mathbf{GF}(\neg s_1 \land \mathbf{X} \neg s_1)$ 

• 
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•  $\gamma_1 = \mathbf{GF}(\neg s_1 \land \mathbf{X} \neg s_1)$   
•  $\gamma_2 = \mathbf{GF} \neg s_2$ 

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